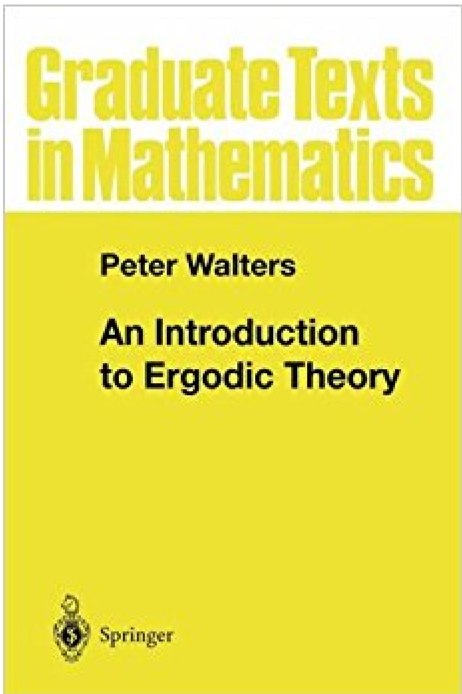
An Introduction To Ergodic Theory (Graduate Texts In Mathematics)



Synopsis

The first part of this introduction to ergodic theory addresses measure-preserving transformations of probability spaces and covers such topics as recurrence properties and the Birkhoff ergodic theorem. The second part focuses on the ergodic theory of continuous transformations of compact metrizable spaces. Several examples are detailed, and the final chapter outlines results and applications of ergodic theory to other branches of mathematics.

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Customer Reviews

This text helped me understand the fundamentals of ergodic theory and allowed me to see how the subject is organized. The explanations of concepts are fairly brief, but the quality of the mathematical exposition is very good. There are several proofs absent however, but it does not greatly diminish the value of the text. In fact, it keeps the length down, so that it is not such a daunting task to finish the book.One can cover the basic concepts of ergodic theory with relatively few classes of examples, which include rotations, endomorphisms, and affine transformations of compact groups preserving Haar measure, as well as Bernoulli and Markov shifts.The preliminary chapter is a quick review of basic measure theory and functional analysis. In the following chapter, ergodicity is described as a form of quantitative recurrence, specifically a measure preserving transformation is ergodic if every set of positive measure A, almost every point of the space eventually gets mapped into A. Later, once the ergodic theorem is proved, this is show to be equivalent to the property that the time average of an integrable function along almost every orbit converges to its integral over the whole space. The last two sections introduce mixing and weak mixing, which give stronger forms of recurrence than ergodicity.Chapter two introduces basic equivalence relations between measure preserving systems, which include isomorphism, conjugacy, and spectral isomorphism. The following chapter deals with the class of measure-preserving transformations with discrete spectrum, for which the eigenvalues of the associated unitary operator on L^2 completely determine the conjugacy class.Chapter four develops the fundamentals of entropy theory, taking no shortcuts to this extremely important subject. The entropy of a measure-preserving system is a numerical invariant which quantifies the asymptotic information generated by the system. Two systems which are conjugate are shown to have the same entropy. This was the breakthrough that allowed a proof of what was before the difficult problem of determining, for example whether or not the Bernoulli shifts (1/2,1/2) and (1/3,1/3,1/3) are conjugate, since they have different entropy. The chapter concludes with an introduction to the K-property and the Pinsker partition.The next part of the book begins with topological dynamics, where recurrence is studied from the continuous perspective. This gives a qualitative as opposed to quantitative description of the long term behavior of the system. It goes on to discuss invariant measures for continuous maps, as well as unique ergodicity. Next we are introduced to the topological counterpart of entropy, which is called topological entropy. It characterizes the exponential growth of the topological complexity of the orbit structure as a single number.The remainder of the book goes on to develop relationships between topological and measure-theoretic entropy, beginning with the variational principle. This fundamental result states that the topological entropy is the supremum of measure-theoretic entropies, where the supremum is taken over all invariant probability measures. It goes on to discuss measures of maximal entropy, the distribution of periodic points, topological pressure, and equilibrium states.The last chapter mentions the multiplicative ergodic theorem, which is a fundamental result in the theory of nonuniformly hyperbolic dynamical systems on manifolds. There are a few other modern developments that could have made it into the book, but I think anyone interested enough would be able to find the appropriate literature.

This was my first exposure to ergodic theory, other than what one picks up here and there in connection with other subjects.The content is summarized in another review, so no need to do it again.The book requires little previous knowledge of probability theory and of measure theory, but it is of course helpful if one has some.As I had no previous exposure of topology, this came a bit harder, but if one has some intuitive understanding, it is enough to understand most proofs and remarks.The proofs themselves are neither too easy nor too complicated, just the right level, it seemed to me. (I am a "hobby mathematician", so to the real student, it should not be hard)The book contains a number (I counted about 30) of misprints, mostly not serious.(I read an authorized Chinese reprint, of course English language!, so maybe the original is still better....)All in all, I can recommend this book.

I think this book is necessary for anyone who wants to study Ergodic Theory: you can find in it all the fundamental elements.Just notice that it requires a good mathematical skill. Reading and understanding it is not always an easy task!